Math 5C Test 3 v2 – Fall 2022

Follow Instructions given on Canvas.

(1) Evaluate $\int_0^{\pi/2}\int_0^{\sqrt{x}}\int_0^{\sin x} \sqrt{x}\, dz\, dy\, dx$ = $\int_0^{\pi/2}\int_0^{\sqrt{x}} \sqrt{x}\ z\Big]_0^{\sin x} dy\,dx$ *(7 points)*

$= \int_0^{\pi/2}\int_0^{\sqrt{x}} \sin x \sqrt{x}\, dy\, dx = \int_0^{\pi/2} \sqrt{x}\sin x\ y\Big]_0^{\sqrt{x}} dx$

$= \int_0^{\pi/2} x \sin x\, dx$ $\qquad u = x \quad dv = \sin x\, dx$; $du = dx \quad v = -\cos x$

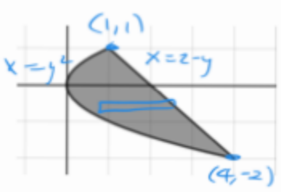
$= -x\cos x\Big]_0^{\pi/2} + \int \cos x\, dx$

$= -x\cos x + \sin x\Big]_0^{\pi/2}$

$= 1 - (0) = \boxed{1}$

---

(2) Evaluate $\iint_D y\, dA$ where D is the region bounded by $x=y^2$ and $y = 2 - x$. *(10 points)*



If dy dx, must split

$\int_{-2}^{1}\int_{y^2}^{2-y} y\, dx\, dy$

$= \int_{-2}^{1} y\, x\Big]_{y^2}^{2-y} dy$

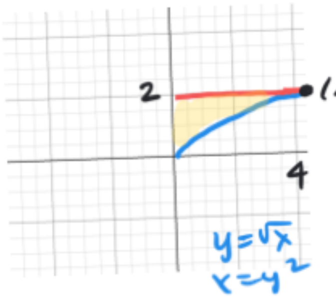
$= \int_{-2}^{1} y(2-y-y^2)\, dy$

$= \int_{-2}^{1} (2y - y^2 - y^3)\, dy$

$= y^2 - \frac{1}{3}y^3 - \frac{1}{4}y^4\Big]_{-2}^{1}$

$= \frac{5}{12} - \frac{8}{3} = -9/4$

(3) Evaluate $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+4}\,dy\,dx$ You may want to reverse the order of integration. *(11 points)*



$$= \int_0^2 \int_0^{y^2} \frac{1}{y^3+4}\,dx\,dy$$

$$= \int_0^2 \frac{y^2}{y^3+4}\,dy \qquad u=y^3+4, \quad du=3y^2dy$$

$$= \frac{1}{3}\int_4^{12} \frac{1}{u}\,du = \frac{1}{3}\ln|u|\Big]_4^{12}$$

$$= \frac{1}{3}(\ln 12 - \ln 4) = \boxed{\frac{1}{3}\ln 3}$$

---

(4) Evaluate $\int_C xy^2\,ds$ where C is the line segment from (-3,0,1) to (4, 2, 5). *(11 points)*

$x = -3+7t$
$y = 2t$
$z = 1+4t$
$0 \le t \le 1$

$ds = \sqrt{49+4+16}\,dt = \sqrt{69}\,dt$

$$\int_0^1 (-3+7t)(4t^2)\sqrt{69}\,dt$$

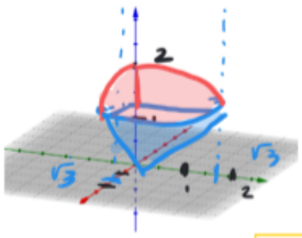
$$= 4\sqrt{69}\int_0^1 (-3t^2+7t^3)\,dt = 4\sqrt{69}\left(-t^3+\frac{7}{4}t^4\right]_0^1$$

$$= 4\sqrt{69}\left(-1+\frac{7}{4}\right) = 4\sqrt{69}\left(\frac{3}{4}\right) = 3\sqrt{69}$$

(5) SET UP BUT DO NOT EVALUATE: integrals as specified to find the volume enclosed above the cone $z = \sqrt{\frac{1}{3}\left(x^2+y^2\right)}$ and inside the sphere $x^2 + y^2 + z^2 = 4$ in the first octant. In each part, sketch the necessary projection

(24points)

a) Sketch the solid



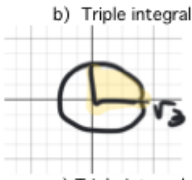
Intersection

$$4-x^2-y^2 = \frac{1}{3}(x^2+y^2)$$

$$4x^2+4y^2=12$$

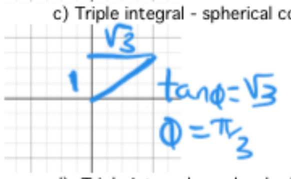
$$x^2+y^2=3,\ z=1$$

b) Triple integral - cylindrical coordinates.



$$\int_0^{\pi/2}\int_0^{\sqrt{3}}\int_{r/\sqrt{3}}^{\sqrt{4-r^2}} dz\, r\, dr\, d\theta$$

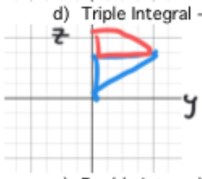
c) Triple integral - spherical coordinates.



$$\int_0^{\pi/2}\int_0^{\pi/3}\int_0^{2} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

d) Triple Integral – order dx dz dy

7



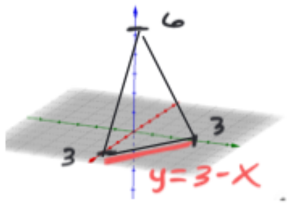
split

$$\int_0^{\sqrt{3}}\int_1^{\sqrt{4-y^2}}\int_0^{\sqrt{4-y^2-z^2}} dx\, dz\, dy + \int_0^{\sqrt{3}}\int_{y/\sqrt{3}}^{1}\int_0^{\sqrt{3z^2-y^2}} dx\, dz\, dy$$

e) Double integral- order dydx

$$\int_0^{\sqrt{3}}\int_0^{\sqrt{3-x^2}} \left(\sqrt{4-x^2-y^2} - \sqrt{\tfrac{1}{3}(x^2+y^2)}\right) dy\, dx$$

(6) Evaluate $\iint_S xz\,dS$ where S is the portion plane 2x+2y+z=6 in the first octant.



surface $z=6-2x-2y$

$$dS=\sqrt{z_x^2+z_y^2+1}\,dA$$
$$=\sqrt{4+4+1}\,dA=3\,dA$$

$$\iint_S xz\,dS$$
$$=\int_0^3\int_0^{3-x} x(6-2x-2y)\,3\,dA$$
$$=3\int_0^3\int_0^{3-x}(6x-2x^2-2xy)\,dA$$
$$=3\int_0^3 \left[6xy-2x^2y-xy^2\right]_0^{3-x}dx$$
$$=3\int_0^3\left(6x(3-x)-2x^2(3-x)-x(3-x)^2\right)dx$$
$$=3\int_0^3\left(9x-6x^2+x^3\right)dx$$
$$=3\left(\frac{9}{2}x^2-2x^3+\frac{1}{4}x^4\right]_0^3$$
$$=3\left(\frac{81}{2}-54+\frac{81}{4}\right)=\boxed{81/4}$$

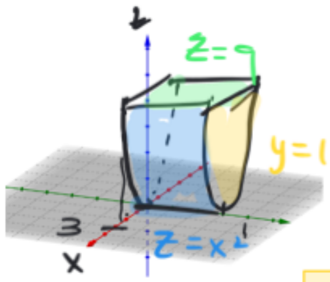
(7) Check all the boxes that apply. A function of three variables might appear in which of the following types of integrals? (6 points)

	Single Integral
	Double Integral
✓	Triple Integral
✓	Line Integral
✓	Surface Integral.

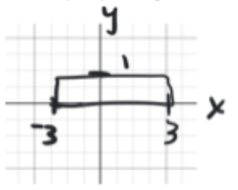
Domain must be in $\mathbb{R}^3$ - can be a
← solid
← curve
← surface

(8) SET UP ONLY :Find the volume of the solid bounded by the surface $z=x^2$ and the planes $y=0$, $y=1$, and $z=9$ according to the following directions. (sketch the solid). In each part, sketch the necessary projection

(20 points)

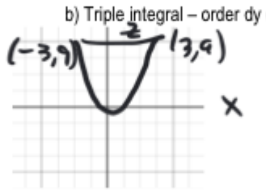


a) Triple integral – order dz dx dy



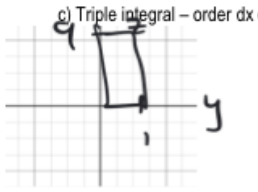
$$\int_0^1 \int_{-3}^{3} \int_{x^2}^{9} dz\, dx\, dy$$

b) Triple integral – order dy dx dz



$$\int_0^9 \int_{-\sqrt{z}}^{\sqrt{z}} \int_0^1 dy\, dx\, dz$$

c) Triple integral – order dx dy dz



$$\int_0^9 \int_0^1 \int_{-\sqrt{z}}^{\sqrt{z}} dx\, dy\, dz$$